force is also indicated. Figure 3 shows the comparisons of the predicted and measured impingement forces for various geometric and flow conditions. It is evident that good correlation between the analytical technique and experimental data has been obtained at all levels of force (stages in close proximity as well as large separation distances). Consequently, this analytical technique should be useful in estimating plume impingement forces during tandem stage separation.

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Taylor Instability in the Shock Layer on a Jovian Atmosphere Entry Probe

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THE forebody of a probe vehicle entering the Jovian atmosphere experiences large radiative heating loads. Ablative heat shields, composed typically of carbon, are contemplated as the only practical means to protect the payload from this heating. For much of the entry the ablation rate because of these heat loads is large enough to produce a phenomenon that has been termed "massive blowing" in which the efflux velocity of ablation products is so great that the boundary layer, normally found at the surface of the body, is actually forced off the body. As a result, a shear layer, which forms the interface between the ablation products and shock-heated freestream gas, is found an appreciable distance from the surface as indicated schematically in Fig. 1. The gas on the freestream side of the layer is fast-moving and relatively hot, has low molecular weight, and is intensely radiating. Conversely, the gas on the vehicle side is more slowly moving, is relatively cool, has moderate molecular weight, and is absorbing much of the radiation that enters from the high-temperature side. In fact, this radiative blockage is extremely important in that it reduces substantially the amount of heat shielding required for the vehicle to survive the entry. Since the pressure difference across the shear layer is insignificant, the disparate temperatures and molecular weights lead to a large density difference—typically a factor of 10—with the higher density occurring in the ablation products. This density difference and the large decelerations during entry constitute the conditions for Taylor instability[1] at this interface. The first purpose of this Note is to examine the severity of the Taylor instability relative to the dynamical instability that is normally present because of the difference in velocity across the shear layer (shear-layer instability). Both instabilities cause growth of initially small disturbances at the interface. In the Jovian entry situation these disturbances are of concern because if there is time for them to grow significantly the result could be severe mixing between ablation products and shock-heated freestream gas. Such mixing might seriously impair or even destroy the effectiveness of the ablation products in protecting the vehicle from radiative heating.

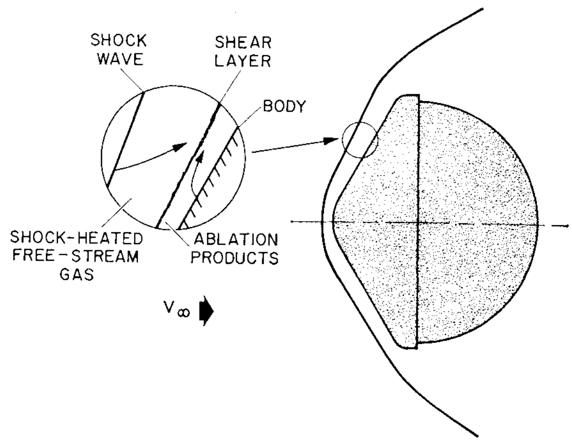

Fig. 1 Sketch of shock-layer flow with "massive blowing."

Since the calculations will show that shear-layer instability is important for a Jovian entry probe, a second purpose of the present Note is to examine approximately the behavior of small bubbles of the shock-heated freestream gas in the ablation products. Such bubbles could be formed as a result of the mixing mentioned above. Because of the deceleration of the vehicle, these bubbles will be bouyant. Thus, calculations will be presented for the distance that the bubbles can travel into the ablation products.

A simplified physical model that will allow assessment of the relative importance of shear-layer and Taylor instabilities is shown in Fig. 2. The quantities ρ and u are interfacial values for density and velocity and are assumed constant for each gas. The radius of the body is R and the external acceleration is a. For inviscid fluids the growth rate of a small disturbance is $\exp(nt)$, where t is time and n is defined as[2]

$$n = \left[\frac{K^2(u_1 - u_2)^2 \rho_1 \rho_2 \coth(Kl_1)\coth(Kl_2)}{[\rho_1 \coth(Kl_1) + \rho_2 \coth(Kl_2)]^2} + \frac{aK(\rho_2 - \rho_1)}{\rho_1 \coth(Kl_1) + \rho_2 \coth(Kl_2)}\right]^{1/2} \quad (1)$$

Here K is the wavenumber of the disturbance, related to its wavelength λ by $K = 2\pi/\lambda$. The first term on the right in Eq. (1) is due to shear instability whereas the second results from Taylor instability. For the high-speed portion of a typical Jovian entry, with a vehicle of 50-cm radius, the char-

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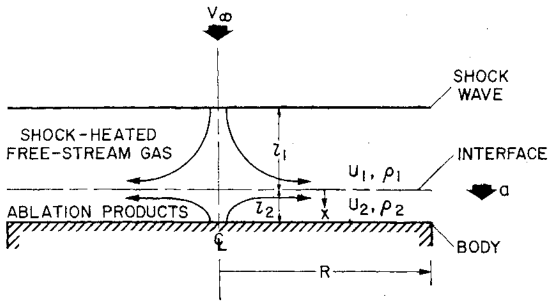

Fig. 2 Simplified physical model of forebody flowfield.

acteristic residence time at the interface is approximately 50×10^{-6} sec; the velocities u_1 and u_2 are typically 1×10^6 and 0.5×10^6 cm/sec; ρ_1 and ρ_2 are typically 10^{-6} and 10^{-5} g/cm^3; and a is approximately 10^6 cm/sec^2 (10^3 Earth g). In terms of λ, and for wavelengths of order l_2 or less, nt becomes

$$nt \sim [10^2/\lambda^2 + 10^{-2}/\lambda]^{1/2} \tag{2}$$

Clearly, from the estimate Eq. (2), Taylor instability is inconsequential when compared with shear-layer instability. Furthermore, the above computation indicates that for all λ of interest the shear-layer instability will cause disturbances initially to amplify very rapidly. This is the normal situation that eventually leads to turbulent flow in a shear layer. For short wavelengths, viscosity moderates and may even act to stabilize this growth. Quantitative results for viscous flow are, however, beyond the scope of this Note.

Since the shear-layer instability is strong, it will cause breakup and mixing of the layer and bubbles of low-density shock-heated gas will thus be trapped within the ablation products. Under the action of the buoyancy force these high-temperature bubbles will move toward the surface of the vehicle before they are swept into the wake by the bulk flow. An estimate of the distance toward the body that a bubble moves can be obtained from the solution of the following simplified equation of motion for the bubble:

$$m(dV/dt) = \underbrace{(\rho_2 - \rho_1)\tfrac{4}{3}a\pi r^3}_{\text{buoyancy force}} \underbrace{- D}_{\text{drag force}} \tag{3}$$

Here m is the mass of the bubble, r its radius, and V its velocity normal to the body. Computations show that bubble Reynolds numbers are low and that is appropriate to compute the drag for Stokes' flow; hence $D = 6\pi\mu_2(V + v_2)r$, where μ_2 is the viscosity of the ablation products (typically 10^{-3} gm/cm sec), and v_2 is the ablation-products velocity normal to the body.

A conservative estimate of the inward distance is obtained by setting $v_2 = 0$ and solving for the distance $x = V_T t$, where V_T is the terminal velocity of the bubble. This velocity is obtained by solving Eq. (3) with $dV/dt = 0$ and is given by

$$V_T = (2/9)(\rho_2 - \rho_1)ar^2/\mu_2 \tag{4}$$

With the preceding numerical values, with $t = 500 \times 10^{-6}$ sec (an overestimate of the residence time in the ablation products), and with $l_2 = 1$ cm, we obtain

$$x/r \approx r(r/l_2)^2(\text{cm})$$

Hence, the small bubbles considered here ($r < 0.1$ cm) move inward a distance less than their radius before they are swept off the body. Such motion is clearly of no concern.

In conclusion, on the basis of the above calculations for a Jovian probe, Taylor instability at the interface between shock-heated freestream gas and ablation products is inconsequential in comparison with shear-layer instability. Furthermore, the motion due to buoyancy of small bubbles of shock-heated freestream gas in the ablation products is also inconsequential. Nevertheless, the state of our knowledge concerning "massively blown" shock layers is completely unsatisfactory as regards both transition criteria and adequate models for turbulent flow. Much work, both experimental and computational, remains to be done on these flowfields.

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Evaluation of Windward Streamline Effective Cone Boundary-Layer Analyses

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Nomenclature

$C_{f\infty}$ = local skin-friction coefficient, $2\tau_w/\rho_\infty V_\infty^2$
L = slant length of sharp cone
M_∞ = freestream Mach number
$\dot{q}_w$ = wall heat flux
Re_∞/ft = unit freestream Reynolds number
$T_{o,\infty}$ = freestream stagnation temperature
T_w = wall temperature
V_∞ = freestream velocity
X_t = surface distance from sharp cone apex to onset of transition location
x = coordinate along body surface measured from apex of sharp cone
α = angle of attack
δ_v = sharp cone semivertex angle
ρ_∞ = freestream mass density
τ_w = wall shear stress

Introduction

ONE of the approximate analysis techniques in common usage among the hypersonic re-entry vehicle designers to estimate windward ray heating rates on slender cones at incidence is the so-called "effective cone" approach in which a zero angle-of-attack calculation is performed on an "effective cone" which has a cone half-angle equal to the physical cone half-angle plus the physical cone angle of attack. Needless to say, the "effective cone" technique does not properly include the effects of crossflow (outflow) as it affects the windward ray boundary-layer structure. The present Note evaluates the results of "effective cone" calculations relative to an exact three-dimensional windward ray analysis for the

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Index categories: Boundary Layers and Convective Heat Transfer—Laminar; Boundary Layers and Convective Heat Transfer—Turbulent; Supersonic and Hypersonic Flow.

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